Evening Meeting

Based on an invited paper presented and discussed at a meeting of the Institution of Structural Engineers, held at the Institution of Civil Engineers, 1 Great George Street, London SW1, on 26 April 2001 at 6pm. Additional material is included.

The London Millennium Footbridge

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Synopsis
This paper starts by describing the structure of the London Millennium Footbridge.

The bridge opened on 10 June 2000. During the opening day unexpected lateral movements occurred as pedestrians crossed the bridge. The paper describes the events of opening day and the research and analysis that were carried out as a result of these movements.

The lateral force exerted by pedestrians on the moving deck surface is found to be related to the movement. The results show that the phenomenon is not related to the technical innovations of the bridge and that the same phenomenon could occur on any bridge with a lateral frequency below about 1.3Hz loaded with a sufficient number of pedestrians. A selection of other bridges, including one road bridge, found to have exhibited the same phenomenon, are listed.

The paper describes the development of a retrofit to control the vibrations. This involves the use of fluid-viscous dampers and tuned mass dampers to achieve damping levels in excess of 20% of critical. The results of tests on the bridge with a prototype installation of a small number of the proposed dampers are presented, which show that the performance of the damping scheme conforms with analytical predictions.

Description of the bridge structure

Introduction

In September 1996 a competition was organised by the *Financial Times* newspaper in association with the London Borough of Southwark to design a new footbridge across the River Thames. Teams consisting of an engineer, an architect and an artist were invited to take part. The competition attracted over 200 entries and was won by the team of Arup (engineers), Foster and Partners (architects) and Sir Anthony Caro (sculptor).

The concept

The location of the bridge was to be in the area between Southwark Bridge and Blackfriars Bridge, but the precise axis for the deck was not prescribed in the competition brief.

Fig 1. View of the aluminium bridge deck looking north

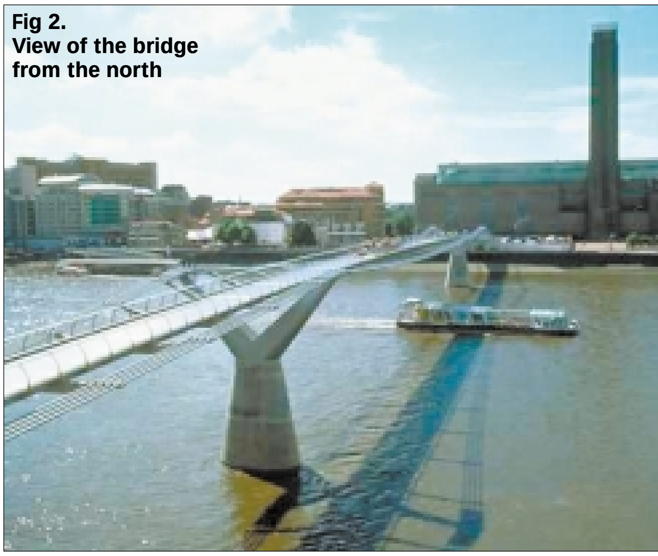


Fig 2. View of the bridge from the north

However, the aims of the bridge and the strong features of the site, the 'corridor' of Peter's Hill on the north bank leading to Saint Paul's Cathedral and the new Tate Modern art gallery on the south, influenced the choice of the bridge axis. Early on the team decided that the axis of Peter's Hill was so strong that the bridge should follow this line directly, giving a clear view straight to St Paul's Cathedral from the bridge deck as a result.

This decision placed the bridge in the zone of influence of the St Paul's height restrictions, allowing a thin sliver for the structure between the navigation channel and the height limit.

Superstructure

General description: The bridge structural diagram is that of a shallow suspension bridge, where the cables are as much as possible below the level of the bridge deck to free the views from the deck. Two groups of four 120mm diameter locked coil cables span from bank to bank over two river piers. The lengths of the three spans are 81m for the north span, 144m for the main span between the piers and 108m for the south span. The sag of the cable profile is 2.3m in the main span, around six times shallower than a more conventional suspension bridge structure.

Fabricated steel box sections, known as the transverse arms, span between the two cable groups every 8m. The 4m wide deck structure comprises two steel edge tubes which span onto the transverse arms. The deck itself is made up of extruded aluminium box sections which span between the edge tubes on each side. Other finishes such as the lighting and handrail are also fixed onto the edge tubes.

The groups of cables are anchored at each bank. Each abutment is founded on a 3m reinforced concrete pilecap anchored by a group of 2.1m diameter reinforced concrete piles. There are 12 piles on the north bank and 16 on the south bank, where the site is constrained and the pilecap shorter in consequence. The river piers themselves comprise a steel 'V' bracket fixed to a tapering elliptical reinforced concrete body which is founded on two No. 6m diameter concrete caissons.

Fig 3.
View of the bridge soffit showing the edge tubes spanning onto the transverse arms every 8m

Cable design: The cables form the primary structure of the bridge and have a very shallow cable profile, as described above. Ribbon bridges have similar shallow profiles, but are typically single spans, allowing the cables to be anchored directly to substantial stiff abutments. The stiff abutments help limit the live load deflections.

The Millennium Bridge has some of the characteristics of a ribbon bridge, but is unusual in having multiple spans. The intermediate river piers are quite slender and cannot provide stiffness comparable to that of a massive abutment. This means the spans interact, making the behaviour of the structure more complex than that of a single span bridge. For example, if only the central span was loaded, the outer spans would deflect upwards.

An initial series of parametric analyses, using the simplest models that could still meaningfully represent the non-linear response of the bridge, helped to understand the bridge behaviour and set realistic targets for parameters such as the cable sizes, and the pier and abutment stiffnesses. The behaviour of each span is driven by the stiffness of the structural system at the extremities of the span. The system providing stiffness is either an abutment or the combination of the cable stiffness of the adjacent span with the stiffness of the pier. These studies quantified the relative stiffnesses of the adjacent span cables and of the pier structures, demonstrating that the cable stiffness provided about 80% and the piers 20% of the total stiffness at such a termination. Hence the main restraint stiffness to the central span came from the outer span cables and from the abutments, not directly from the piers. It was therefore important that variations in the abutment stiffnesses would not result in similar variations of the bridge deflections. The parametric studies enabled limiting foundation stiffnesses to be selected to achieve this requirement.

A shallow cable profile necessarily results in large cable tensions. The deadload of the bridge is 2t/linear m along the bridge axis. The resulting total dead load cable tension is 22.5MN. In a conventional bridge, the deck is braced to form a girder, which resists lateral wind load. In the case of the Millennium Bridge, the structural system in both vertical and horizontal directions is similar, as it derives most of its stiffness from the tension stiffness of the cables. With the stiffness of the structure selected to limit deflection under vertical loading, which is higher than wind loading, the Millennium Bridge has sufficient tension stiffness for the deck not to require any additional wind resisting structure. The dynamic characteristics of the bridge demonstrate that its lateral stiffness is in the same range as other footbridges of similar span. This is borne out by Fig 4 which shows a survey of other footbridge lateral frequencies versus their spans. It can be seen that the Millennium Bridge frequencies are in the same range as the other bridges.

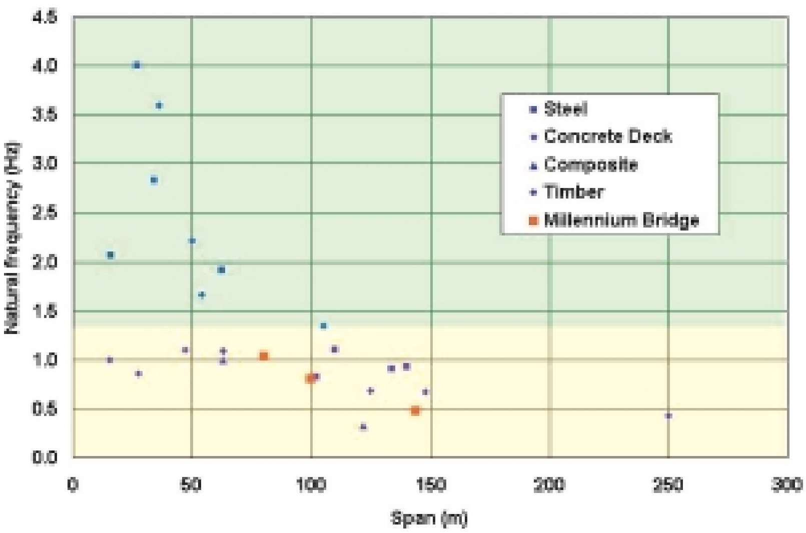


Fig 4.
Natural frequencies of lateral modes for footbridges

The cables are set wide apart in plan and well beyond the width of the deck. This was chosen to increase the torsional stiffness of the bridge.This geometry has two advantages. Firstly the torsional deflection due to an asymmetric live load across the width of the bridge is minimised. Secondly, the increased torsional stiffness helps separate the torsional and translational frequencies of the structure. This improves its aerodynamic performance and is described further below

Articulated deck: The tension stiffness is such that a continuous deck structure would contribute little extra stiffness to the system, whilst attracting significant parasitic forces in the deck members due to the cable movements. The decision was therefore taken to introduce articulated sliding joints at regular intervals along the length. Parametric studies were carried out to optimise the spacing of the joints and a typical spacing of 16m was selected. This means that the deck structure can be very simple, a series of prefabricated 'trestles' 16m long placed on the cables, which also facilitates the construction of the bridge deck. In this way, the overall depth of the deck structure is kept to an absolute minimum.

South needle: The deck bifurcates at the south end of the bridge; this area is known as the needle because of its form in plan. Initial studies considered propping the deck from the ground in that area, whilst maintaining some support from the cables. Nonlinear analysis showed that combining props and cable support would generate very large forces in the props as they tried to restrain the cable movements. A solution with props only was also considered but was rejected as it introduced a different way of supporting the bridge deck before the end of the bridge. After further iterations, it was decided that the only solution which retained the structural concept for the whole length of the bridge was to cantilever the deck from the cables on each side, using torsion in the deck edge tubes to resist the overturning moment and to control the rotation of the deck. Accordingly, the edge tubes were thickened and increased to ovoid sections in that area.

Piers: The pier body has a tapering elliptical cross section, cast in C60 reinforced concrete. The pier formwork was fabricated in three sections, each approximately 5 m high, constructed off site and brought into position by barge. A series of sleeved 75mm diameter high strength steel bars cast into the top lifts is used to connect the steel pier 'V' brackets onto this concrete body.

The design of the pier was driven by two main objectives. Firstly, the piers have to withstand ship impact. Secondly, the pier form must be such as to minimise its impact on the river hydraulics.

The design loads and criteria for ship impact resulted from a study of the traffic on the Thames and the river characteristics at the location of the bridge.

The River Thames is a busy waterway, both for commercial and for tourist traffic. Various ships navigate the Thames at the Millennium Bridge site, from recreational craft and sightseeing craft to rubbish barges and sand and aggregate carriers. The biggest ship at the time of analysis was the 55m-long *Tracy Bennett*, with 1210 DW.

The precise nature of the river traffic remains, nevertheless, relatively undefined. In order to establish river use more accurately, Arup commissioned a round-the-clock survey over a period of a week, during which every vessel passing under Southwark Bridge was recorded, along with its size, displacement and position in the river channel. It was found that there are on average 400 boats a day which pass the bridge site. The resultant raw data was processed in order to take account of seasonal variations in shipping. It was then used to establish the impact forces to the piers via the probabilistic method set out by the AASHTO code[1]. Future water level changes due to sea level rise were considered in order to define the height at which the force would be applied, and the risks of impact occurring.

The piers have been designed both to withstand collapse under the load set out in the competition brief, and to resist, with the need for remedial repairs, a more precise load calculated using the AASHTO probabilistic approach. The deck structure and cables are not susceptible to ship impact over the majority of the span. A risk assessment carried out for the south span established the height for the cable elevation at the south abutment, and resulted in the design of the south abutment 'wing' structure to anchor the cables 4m above ground level.

Placing piers, cofferdams and construction equipment in a river locally changes the nature of its hydraulics. In order to investigate these effects, Arup commissioned HR Wallingford to construct a 1:100 natural scale physical model to investigate the impact of the Millennium Bridge on the River Thames. Permanent bed protection was placed around the piers and temporary bed protection was placed on the foreshore as a result of the tests after consultation with the Environment Agency and the Port of London Authority. The tests on the piers also confirmed that turbulence around the permanent works was sufficiently low that it would not hamper navigation through the bridge or at the Bankside pontoon.

The cables are locked longitudinally at the top of the pier against the saddles by a series of friction clamps. The steel pier V-brackets are in turn fixed to the concrete body of the pier via the pretensioned high-strength steel bars described above. During construction, the pier V-brackets were left articulated on hydraulic jacks in order to take up final adjustment. The jacks and the articulation of the pier V-bracket to the concrete body can be reinstated at a future date enabling the alignment of the pier to be modified to retension the cables.

South abutment: The cables are anchored down to the 3m pilecap on the south side by a steel 'strut and tie' on each side of the concrete ramp. The strut and tie lie in the same plane as the cable when subject to dead load alone. In this way the out-of-plane forces acting on the abutment structure are due only to the change in cable geometry from live load on the bridge.

North abutment: The cables are anchored on the north abutment at the top of the steps leading up Peter's Hill. The large, predominantly horizontal, forces from the cables transfer via a horizontal reinforced concrete deep beam to vertical shear walls on each side of the steps. These shear walls, 1m thick, carry the loads to the 3m pilecap. In this way the existing space below the steps, used by the City of London school for storage and parking, is maintained.

Substructure

Ground conditions: Ground conditions were investigated by Norwest Holst Soil Engineering Ltd in July and August 1998. Boreholes were sunk at the abutments and at the river pier locations. The stratigraphy comprises Made Ground overlying a sequence of Terrace Gravels, London Clay and the Lambeth Beds.

Abutment foundations: The most significant loads applied to the abutment foundations will be the horizontal force (some 30MN combined dead and live load) from the bridge cables, and the resulting overturning moment. The foundation piles resist the horizontal loading by lateral loading of the soil. The moment is resisted partly by frame action but mainly by vertical push-pull loading of the piles. The design of the foundation system is principally constrained by the requirement to limit horizontal deformations at the cable anchorage due to lateral movement and rotation of the pile cap. A combination of single pile and pile group analyses were carried out using in-house programs to determine the stiffness of the foundation and the forces in the substructure.

An additional consideration is the need to limit the effect of ground movements on the adjacent structures. The horizontal loading will cause some soil pressures to be applied to the piles supporting the neighbouring buildings and the river walls, which they should be able to sustain without distress.

The north abutment site was excavated down to just below the pile cap soffit level by the Museum of London Archeological Service to investigate its archaeological potential. On three sides of the excavation the surrounding soil was supported by a temporary king post wall system, the former Swiss Bank Building basement wall being on the fourth side. The excavation was temporarily backfilled prior to piling to allow access for the piling rig. The foundations comprise a group of 12 bored cast *in situ* concrete piles, 2.1m diameter and 28m long. The piles were constructed from C40 concrete and reinforced at the top with 44 T50 bars, reducing in number down the length of the piles. The length of the piles and reinforcement were sized primarily to provide the desired stiffness response of the foundation.

The south abutment site was also archaeologically excavated. As there was more room on the south side of the river, battered side slopes could be formed and so the piles could installed from the base of the dig. The foundations comprise a group of 16 bored cast *in situ* concrete piles, 2.1m diameter and 28m long with a 3m deep pile cap.

The abutment piles will tend to apply a pressure to the back of the river walls as they are pulled forwards by the force from the cables. This could lead to adverse effects on the walls unless specific protection measures are taken. The original proposal to avoid loading the river walls comprised a sleeving arrangement around the front row of piles at each abutment, which left a 75mm clear void between the piles and the surrounding soil behind the river walls. The outer sleeve comprised a 2.5mm-thick ribbed steel tube, which was designed to be flexible enough to distort under loading from the piles behind. An alternative system was developed during discussion between Arup, Bachy-Soletanche and Monberg Thorsen McAlpine, whereby the front two rows of piles are surrounded by an annulus filled with a weak, cement-bentonite slurry. The slurry had to be sufficiently dense and strong to support the soil, but soft enough to transfer only a limited amount of load to the ground as the piles deform.

The riverside face of the pile caps are separated from the soil by Cellcore, a compressible grillage formed from expanded polystyrene, again to avoid loading the river wall. The other three sides of the pile caps are separated from the soil by 200mm-thick expanded polystyrene board. Each pilecap contains around $1000m^3$ of reinforced C40 concrete, placed in one single pour on the south abutment and two on the north due to traffic constraints in the area.

Pier foundations: The bridge is supported by two piers in the

Fig 5. (below) Piling on the south abutment
Fig 6. (below right) View of the river piles cofferdam

River Thames. The foundations for each pier comprise a pair of 6m diameter caissons, dug to some 18m below river bed level within a sheet pile cofferdam, connected via a 3m deep pilecap. The caisson system was proposed by the contractor as an alternative to the original driven steel pile supports and refined by the design team to ensure that adequate lateral stiffness would be provided. The pier foundations have to withstand a 35.9MN head on and skew ship impact without significant permanent displacements as well as providing adequate longitudinal stiffness to the cables. The effects of sheet pile withdrawal were also studied.

The design of the caissons was carried out using a 3-D finite element analysis which modelled the structure and the soil by 40 000 brick elements. The non-linear behaviour of the soil was simulated to represent its stiffness at relatively small movements in service as well as the large displacements which would occur during a ship impact.

The caisson construction made use of standard water industry tunnel lining precast segments to excavate a vertical tube into the underlying clays by a combination of mini-excavator and clay spades. The segments were grouted to ensure good contact with the surrounding soil. The caissons were subsequently backfilled with reinforced concrete. During excavation the soil beneath the shafts was drained using pre-drilled relief wells to reduce the likelihood of base failure, with monitoring carried out using arrays of vibrating wire piezometers.

Dynamic characteristics of the bridge

Analysis method: Any analysis of the bridge has to allow for the dominant tension stiffness. The modeshapes and natural frequencies were calculated using the Arup GSA program, which has the capability to use a combined normal and geometric stiffness matrix in an eigen analysis. The modeshapes are close to sinusoidal, with frequency related to tension, wavelength and mass for translational modes, or inertia for torsional modes. The modes tend to only have a significant response in one span, allowing them to be characterised as, say, the second torsional mode of the south span.

Wind excitation: The influence of the wind on the bridge has been examined in terms of wind load, aerodynamic stability, buffet movements and environment for bridge users. The first of three sectional model wind tunnel tests took place during the competition to confirm the feasibility of the concept. Further tests during scheme design established design wind loads and the parametrics governing stability, and confirmatory tests were carried out once the geometry had been finalised. These studies were performed at RWDI's laboratories in Canada with 1:16 scale sectional models. The position of the cables relative to the deck, and the mass, stiffness and polar moment of inertia were varied in order to model different sections along the bridge.

The bridge was shown to be aerodynamically stable in wind speeds up to and beyond a 10 000 year return period wind event. The slim depth and round edges of the cross section contribute to the bridge stability. The separation of the torsional and translational frequencies as described above minimises the interaction between those modes, preventing flutter instability. The cable arrangement contributes positive aerodynamic damping of torsional motion. Significant vortex shedding excitation does not occur.

The response to wind buffeting was also evaluated during the design period. The buffeting response of the bridge is primarily vertical and mainly occurs in the lowest modes. Because there are limited data on acceptable levels of vibration for frequencies below 1Hz, motion simulator tests were carried out at the Institute of Sound and Vibration Research at the University of Southampton to confirm the acceptability of the predicted motions. Subsequent monitoring of the built bridge has validated the predicted levels of buffet-induced movement.

Pedestrian excitation based on BS 5400: During design, a modified BS 5400 approach was used for assessment of the response to vertical pedestrian excitation, using a higher input force than is recommended in the code. To take into account the effects resulting from the inclined cables, including the coupling of lateral and torsional movements, the lateral and torsional response to eccentrically applied vertical loads was also assessed.

Whilst it is possible to calculate the response to pedestrian excitation in a manner consistent with BS 5400 using modal time history analysis, the bridge has many modes in the frequency range of pedestrian input, hence time history analysis is somewhat impractical. By approximating the modeshapes of the bridge to sinusoids, it is possible to obtain a closed form solution for the likely peak response in each mode. This method allowed all pedestrian excitable modes to be assessed. The responses have been calculated both in terms of acceleration and response factor R, this being the ratio of acceleration to the acceleration that a human can just perceive at that frequency.

The peak response in the vertical modes is associated with the north span. This is because the north span is the shortest, so its modes have the least modal mass. The peak acceleration was 19 milli-g, the peak response factor R = 18.

Below about 3Hz, humans are more sensitive to horizontal than vertical accelerations. Torsion modes give rise to both vertical and horizontal response. On the Millennium Bridge, about 50% of the mass of the bridge superstructure is in the cables and cable clamps. Spacing the cables wide apart results in the torsion modes having relatively high modal mass. The torsion modes also have relatively low excitability, because the deck moves less than the cables.

This means that the response in torsional modes is generally lower than in vertical modes, even when the sensitivity to horizontal acceleration is taken into account. The north span is an exception. Where the bridge meets the Peter's Hill Steps, the cables cannot be spaced much wider apart than the deck width, or they would obstruct the stairs. The benefit of relatively high modal mass and low excitability is reduced, resulting in higher peak vertical and horizontal accelerations of 17 and 11 milli-g, combining to give a response factor of R = 26.

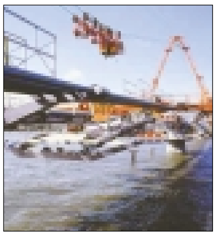


Fig 7 & 8. View of river piers showing 'Highline' used to install cables

Testing with a few people took place on the bridge in May 2000 prior to the opening and confirmed the calculated accelerations.

Construction

Construction began on site with the archeological excavation of each abutment in late 1998. The enabling works were carried out by Balfour Beatty.

A Joint Venture of Monberg Thorsen and Sir Robert McAlpine won the contract for the Main Works and started piling on site in April 1999. The superstructure began to be erected at the beginning of 2000, and the bridge opened on 10 June 2000.

Opening day

It is estimated that between 80 000 and 100 000 people crossed the bridge during the first day. Analysis of video footage showed a maximum of 2000 people on the deck at any one time, resulting in a maximum density of between 1.3 and 1.5 people per square metre.

Unexpected excessive lateral vibrations of the bridge

Fig 9. Opening day

occurred. The movements took place mainly on the south span, at a frequency of around 0.8Hz (the first south lateral mode), and on the central span, at frequencies of just under 0.5Hz and 1.0Hz (the first and second lateral modes respectively). More rarely, movement occurred on the north span at a frequency of just over 1.0Hz (the first north lateral mode).

Excessive vibration did not occur continuously, but built up when a large number of pedestrians were on the affected spans of the bridge and died down if the number of people on the bridge reduced, or if the people stopped walking. From visual estimation of the amplitude of the movements on the south and central span, the maximum lateral acceleration experienced on the bridge was between 200 and 250 milli-g. At this level of acceleration a significant number of pedestrians began to have difficulty in walking and held onto the balustrades for support.

No excessive vertical vibration was observed.

The number of pedestrians allowed onto the bridge was reduced on Sunday 11 June, and the movements occurred far more rarely. On 12 June it was decided to close the bridge in order to fully investigate the cause of the movements.

Investigations

Description of the phenomenon of synchronous lateral excitation

The movement at the Millennium Bridge was clearly caused by a substantial lateral loading effect which had not been anticipated during design. The loading effect has been found to be due to the synchronisation of lateral footfall forces within a large crowd of pedestrians on the bridge. This arises because it is more comfortable for pedestrians to walk in synchronisation with the natural swaying of the bridge, even if the degree of swaying is initially very small. The pedestrians find this makes their interaction with the movement of the bridge more predictable and helps them maintain their lateral balance. This instinctive behaviour ensures that footfall forces are applied at the resonant frequency of the bridge, and with a phase such as to increase the motion of the bridge. As the amplitude of the motion increases, the lateral force imparted by individuals increases, as does the degree of correlation between individuals. It was subsequently determined, as described below, that for potentially susceptible spans there is a critical number of pedestrians that will cause the vibrations to increase to unacceptable levels.

Although a few previous reports of this phenomenon were found in the literature, none of them gave any reliable quantification of the lateral force due to the pedestrians, or any relationship between the force exerted and the movement of the deck surface.

Research and analysis

Three priorities were identified after opening day:

(1) To compare the dynamic properties of the built structure to the analytical predictions.
(2) To quantify the forces that were being exerted on the structure by the pedestrians.
(3) To design a retrofit to install on the bridge that would reduce these movements to acceptable levels.

Measurement of the existing structure

Extensive modal testing of the bridge was carried out following the closure and the results were compared to the design predictions for the modal properties.

The damping for each mode was measured at around 0.6 to 0.8% of critical. This showed the design assumption of 0.5% of critical to be conservative. The measured frequencies of the modes which had been observed to vibrate were within 10% of the values predicted in the design analysis, indicating that the design models had realistically predicted the actual properties of the bridge.

The conclusion from these tests on the built structure was that the initial analytical model of the bridge structure was valid. This meant that the movements were due to an unpredicted external force, rather than unpredicted or miscalculated bridge dynamic characteristics. It also meant that the same structural analytical model could be used for the development of a retrofit to reduce the movements.

Testing and research

A programme of research was undertaken rapidly to establish the lateral forces induced by crowds of pedestrians. No relevant research was found in the literature on the influence on pedestrian forces of lateral motions of the surface on which the people are walking. Research was conducted through laboratory tests and crowd tests on the Millennium Bridge itself.

Laboratory tests: Tests involving pedestrians walking on moving platforms were carried out at the University of Southampton and at Imperial College, London. The intention of these tests was to establish the following:

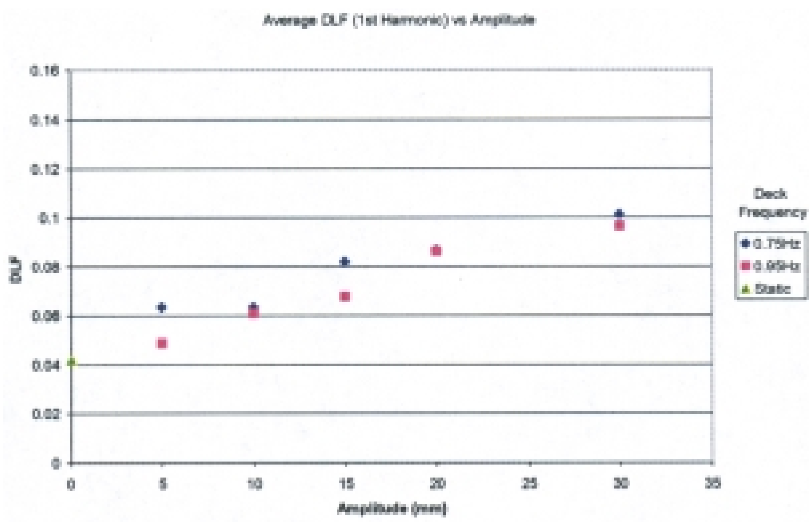
- How the lateral force an individual imparts to a swaying platform on which he is walking varies with the frequency and amplitude of the sway.
- The probability that a pedestrian will synchronise his footfall rate to the frequency of the swaying platform (lock-in), as a function of the frequency and amplitude of the sway.

The tests at the University of Southampton involved a person walking 'on the spot' on a small shake table. The tests at Imperial College involved persons walking along a specially built, 7.2m-long platform which could be driven laterally at different frequencies and amplitudes.

Each type of test had its limitations. The Imperial College tests were only able to capture 7 – 8 footsteps, and the 'walking on the spot' tests, although monitoring many footsteps, could not investigate normal forward walking. Neither test could investigate any influence of other people in a crowd on the behaviour of the individual being tested.

Nonetheless, useful results were obtained, which enabled the design of the retrofit to be progressed. Fig 10 shows the mean first harmonic lateral dynamic load factors (the ratio between the lateral dynamic force and the weight of the pedes-

Fig 10. Average dynamic load factor versus amplitude of movement



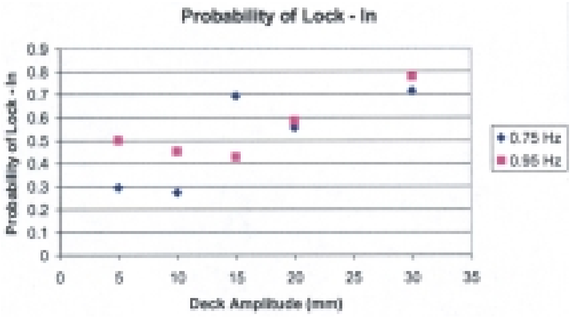


Fig 11. Estimated probability of Lock-in versus Amplitude

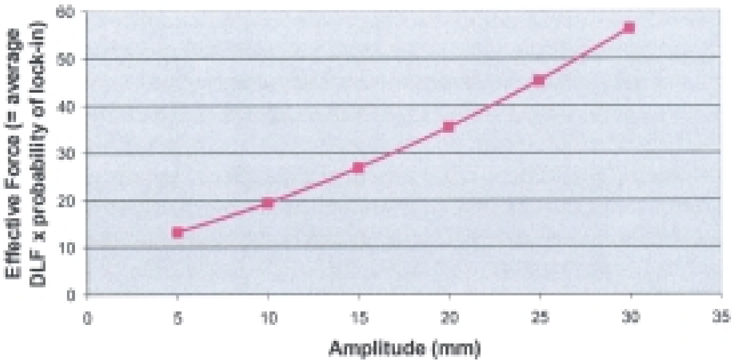


Fig 12. Effective force versus amplitude of movement

trian) for walking as a function of deck amplitude as measured at the Imperial College tests. Also shown in Fig 11 is the estimated mean probability that individuals will synchronise their footfall rates to the swaying rate of the platform.
The overall correlated force from a large crowd can be estimated as the force per person multiplied by the number of people locked-in. Expressed alternatively, the effective correlated force per person in a large crowd is the lateral force per person multiplied by the probability that the individual is locked-in. Fig 12 shows the effective force per person estimated from the Imperial College tests as a function of bridge velocity. It can be seen that the force appears to increase steadily with increasing deck motion.

Tests on Millennium Bridge: The results of the laboratory tests provided information which enabled the initial design of a retrofit to be progressed. However, the limitations of these tests was clear and it was felt that the only way to replicate properly the precise conditions of the Millennium Bridge was to carry out crowd tests on the bridge deck itself. These tests could incorporate factors not possible in the laboratory tests, in particular:

- Tests with groups of people would include the effects of 'psychological' crowd-related factors that might influence the correlation between individuals.
- The degree and 'quality' of lock-in over a large number of footsteps of proper walking would be replicated.

The first of these was carried out with 100 people in July 2000. The results of these tests were used to refine the load model for the pedestrians.

A second series of crowd tests was carried out on the bridge in December 2000. The purpose of these tests was to further validate the design assumptions and to load test a prototype damper installation. The test was carried out with 275 people.

Determination of pedestrian loads

Pedestrian tests
The loads generated by pedestrians were investigated in a variety of tests. The data reported here was collected in the tests carried out on the bridge in December 2000 (Fig 13.). In these tests people circulated anticlockwise around two marker poles spaced apart on the centreline of the deck. By varying the position of the poles between tests, the density of the crowd and its effectiveness at exciting a particular mode via the modeshape could be altered. As each test progressed the number of people was slowly increased by adding people in small groups. Accelerometers and video cameras were used to record what happened.

Fig 13. Tests carried out on bridge in December 2000

Theory
The pedestrian loads were determined using the following theory. The notation used is:

A, a	= modal acceleration
C	= modal damping
c	= critical damping ratio
c_e	= equivalent negative damping ratio
c_{eff}	= effective damping ratio $c+c_e$
D	= modal damping force
E	= modal energy
E(x)	= expected value of variable x
F, f	= force
f	= natural frequency (Hz)
Fe	= correlated modal excitation force
H	= transfer function
K	= modal stiffness
k	= lateral walking force coefficient (Ns/m)
L	= length of span
M	= modal mass
N	= number of people
N_L	= limiting number of people
p(x)	= normal probability density function
Pd	= power absorbed by damping
Pe	= excitation power
t	= time
V	= modal velocity
V_{local}	= local deck velocity
y	= modal displacement
αF_1	= correlated single person force
δ	= log decrement damping
ϕ	= modeshape
μ	= mean value
ρ	= correlation coefficient
σ	= standard deviation
ω	= natural frequency radian/s

Magnitudes of periodic variables are denoted by capital letters, while small letters denote instantaneous values. For example, over a short period of time, the acceleration time history could be described by $a = A\sin\omega t$.

The response of a system to a sinusoidally varying input force depends on the relative frequency of the force to the natural frequency of the system. The response to a relatively low frequency input is stiffness dominated, with the displacement in phase with the force. The response to a relatively high frequency input is inertia dominated, with the acceleration in phase with the force. By far the greatest response occurs when the forcing frequency is close to the natural frequency of the system. The stiffness and inertia forces tend to cancel, the resonant response is limited by damping, with the velocity in phase with the force.

The strong lateral response of the Millennium Bridge was caused by resonance. The ability of pedestrian to excite the bridge can be quantified by considering how much power the pedestrians put into the system. Power equals the product of force × velocity. A representative average power can be obtained by taking the average of this product over a complete vibration cycle. If the force and velocity are both sinusoidal, the power will depend on the phase difference between them. Only the component of force in phase with the velocity puts power into the system. Components of force at +/- 90° effectively add stiffness or mass to the system, modifying the natural frequency. A component of force at 180° to the velocity adds damping. Taking Fe as the component of force in phase with the velocity (the correlated excitation force), the average power input by the pedestrians is:

$$Pe = \frac{1}{2} FeV$$

The power absorbed by the dampers is:

$$Pd = \frac{1}{2} DV$$

The resultant power input equals the rate of change of modal energy against time:

$$Pe - Pd = \frac{d}{dt} E = \frac{d}{dt}\left(\frac{1}{2} MV^2\right) = MV \frac{dV}{dt}$$

Reordering and substituting the power relationships given above:

$$\frac{1}{2} FeV = \frac{1}{2} DV + MV \frac{dV}{dt}$$

$$Fe = D + 2M \frac{dV}{dt}$$

Substituting $A = \omega V$ gives:

$$Fe = D + 2M \frac{1}{\omega} \frac{dA}{dt} \qquad \text{...equation 1}$$

This equation was used to process the test acceleration data, taking the modal damping force D as a nonlinear function of amplitude. The nonlinearity was due largely to stiction in the viscous dampers used in some tests to increase the damping of the bridge. If the damping is assumed to be linear, the modal damping force can be related to the critical damping ratio:

$$C_{cr} = 2\sqrt{MK} = 2\omega M$$

$$D = CV = cC_{cr} V = 2c\omega MV = 2cMA$$

Approximating the rate of change of acceleration by the change over one period:

$$\frac{dA}{dt} \approx \frac{\Delta A}{\Delta t} = \frac{\Delta A}{2\pi/\omega}$$

Substituting these expressions into equation 1:

$$Fe = 2cMA + M \frac{\Delta A}{\pi}$$

While this equation may not be familiar, it can be shown to embody two well-known relationships. Setting ΔA to zero gives the familiar equation for steady state response:

$$Fe = 2cMA$$

$$A = \frac{1}{2c} \frac{Fe}{M}$$

Setting Fe to zero gives the familiar equation for damped decay:

$$2cMA + M \frac{\Delta A}{\pi} = 0$$

$$\frac{\Delta A}{A} = - 2\pi c = - \delta$$

Data processing

To obtain the acceleration in a particular mode, the relevant accelerometer signals were first combined, then digitally filtered to remove content away from the natural frequency. The resulting time history signal was processed to obtain a record of acceleration magnitude at each period. Equation 1 was used to convert this into a record of modal force. The modal force was then divided by the effective number of people involved in the test at that time, to obtain a record of the correlated modal force per person αF_1.

Modal force

Typical histories of vertical and lateral modal force per person are shown in Figs 14 and 15. These relate to the fifth vertical mode of the centre span (CV5) at 1.9Hz and the first lateral mode of the centre span (CL1) at 0.5Hz. The vertical force appears to vary randomly about a zero mean, implying that people are as likely to be attenuating the movement of the bridge as adding to it. In contrast the lateral force varies much less rapidly and is nearly always positive, implying the people act to sustain or amplify the movement of the bridge.

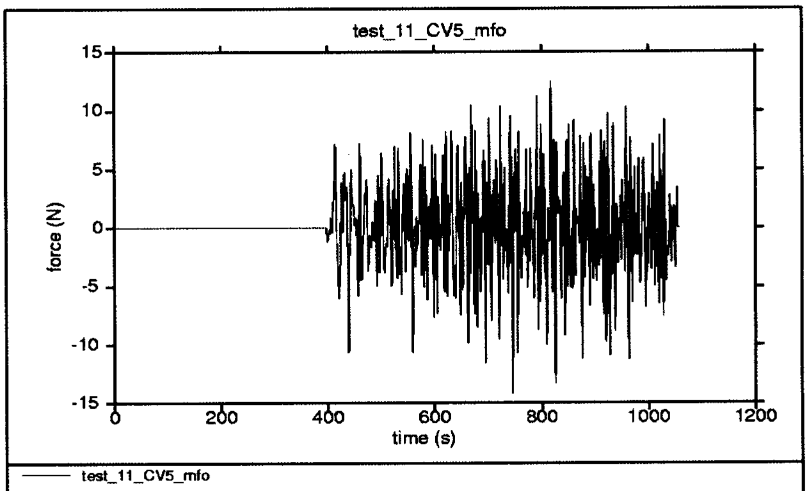


Fig 14. Typical vertical modal force

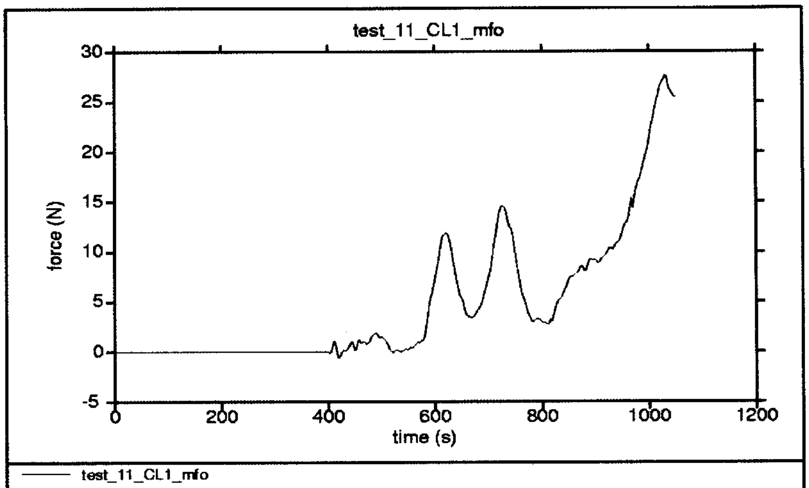


Fig 15. Typical lateral modal force

Correlation with bridge response

Force data was plotted against bridge response to investigate the hypothesis that the lateral force and response were correlated. The modal force was converted into a physical force by dividing by the root mean square modeshape ($1/\sqrt{2}$ for a sinusoid with the whole span loaded). The modal acceleration was converted to a velocity, which was multiplied by the same modeshape factor to obtain a local physical velocity. Typical plots of the correlated physical force per person against local physical velocity are shown in Figs 16 and 17. In these plots there are data points each period, which for the CV5 and CL1 modes are 0.52s and 2.0s.

The difference between the plots is striking. The vertical force shows no correlation with bridge response. In contrast, the lateral force varies almost linearly with the response. The correlation is sustained over time, the rise in lateral force from 20N to 40N occurring over some 60s.

The difference between vertical and lateral behaviour is reinforced by Figs 18 and 19, which show all the CV5 and CL1 data. The correlation can be quantified by the correlation coefficient:

$$\rho_{xy} = \frac{E\left[\left(x - m_x\right)\left(y - m_y\right)\right]}{\sigma_x \sigma_y}$$

The correlation coefficients for the vertical data are insignificant, lying in the range ±0.025. The coefficients for the lateral data vary between 0.34 and 0.73, the higher values occurring for the tests involving higher accelerations.

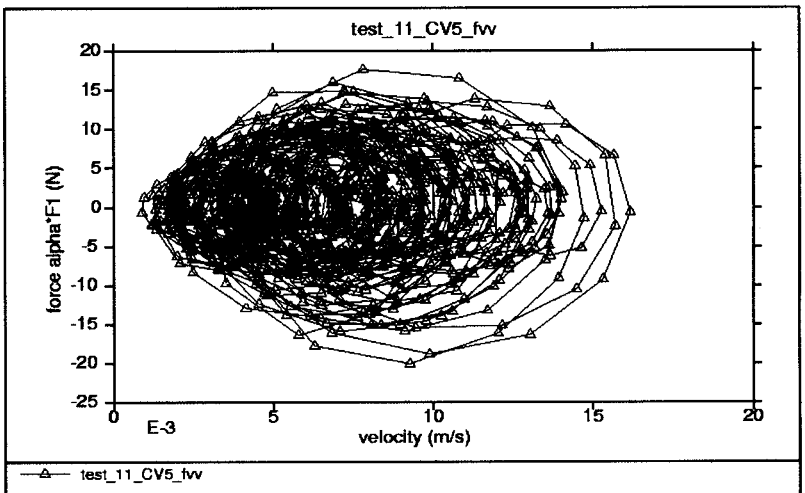


Fig 16. Typical vertical force versus velocity

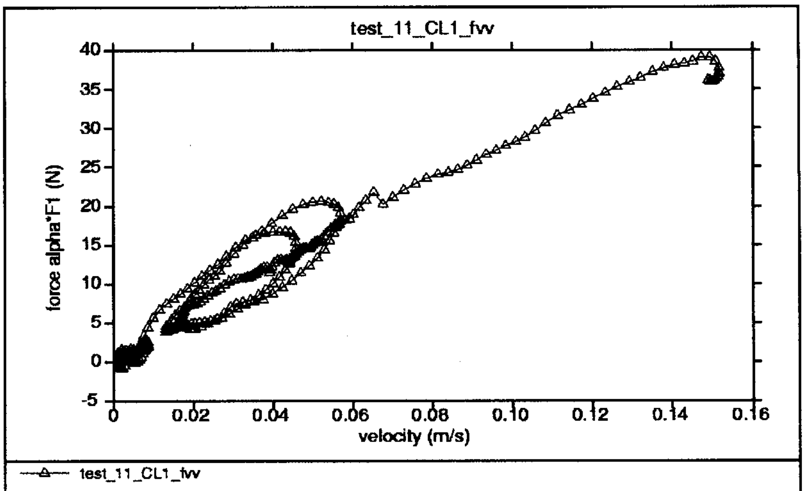


Fig 17. Typical lateral force versus velocity

The Millennium Bridge data provides no evidence that the vertical forces generated by pedestrians are other than random, while the lateral forces are strongly correlated with the lateral movement of the bridge. A possible explanation for this difference is that pedestrians are less stable laterally than vertically, which leads to them being more sensitive to lateral vibration and to modify their walking patterns when they experience such vibration.

One should however be cautious in regarding this as a general conclusion. The problems on the Millennium Bridge involved lateral not vertical vibration. As shown in Figs 18 and 19, the tests produced vertical accelerations of around 35 milli-g, well within vertical comfort limits, while the lateral accelerations were sometimes beyond the 20 to 40 milli-g range desirable for comfort. It is possible that higher vertical accelerations would increase the correlation of the vertical forces.

Components of pedestrian loading

Having quantified the modal force that causes lateral response, it is of interest to consider how the pedestrians create this excitation. A pedestrian generates vertical, lateral and torsional forces, all of which could potentially excite a bridge. The pedestrian vertical forces correspond to each footfall, and typically occur at around 2.0Hz. The coupling of lateral and torsional modes on the Millennium Bridge make it possible for vertical forces to excite lateral response. However, the modes of interest were between 0.5 and 1.0Hz, too low to be excited by the 2.0Hz pedestrian vertical forces.

The lateral and torsional forces both arise from alternate foot forces. They occur at half walking frequency and are likely to have similar phase and levels of correlation. The forces are related in that they are both proportional to the spacing of the feet. It is therefore possible to assess the relative contribution of the two forces using the associated lateral and roll components of the modeshapes. Torsional input is estimated to increase the response of the first lateral mode of the south span, which has the greatest torsional coupling, by 11%. The increases for the first lateral modes of the centre and north spans are 6% and 3%. It was concluded that the lateral component of load was by far the most significant.

Pedestrian vertical loading

Based on the evidence that the vertical pedestrian forces are random, the vertical response should be assessed using random vibration theory. Generally, the expected response y of a linear system can be related to the responses y_i due to individual inputs i by the relationship:

$$E(y^2) = \sum_i E(y_i^2) \qquad \text{... equation 2}$$

Assume that the i-th input force f_i is applied at a frequency ω_i and the system has a natural frequency ω_n, such that the relative frequency $\omega_i/\omega_n = \omega_{ri}$. If the force is applied with a modeshape factor ϕ_i the response y_i can be expressed:

$$y_i = \frac{1/K}{(1-\omega_{ri}^2) + 2ic\omega_{ri}}\phi_i f_i = H_i \phi_i f_i$$

$$E(y_i^2) = H_i^2 E(\phi_i^2) E(f_i^2) \qquad \text{... equation 3}$$

If there are N such inputs with frequencies normally distributed with a mean μ, standard deviation σ and probability density function p, then the expected number of such input forces in the frequency band ω_i to $\omega_i + d\omega$ is:

$$dN = \frac{N}{\sigma} p\left(\frac{\omega_i - \mu}{\sigma}\right) d\omega \qquad \text{... equation 4}$$

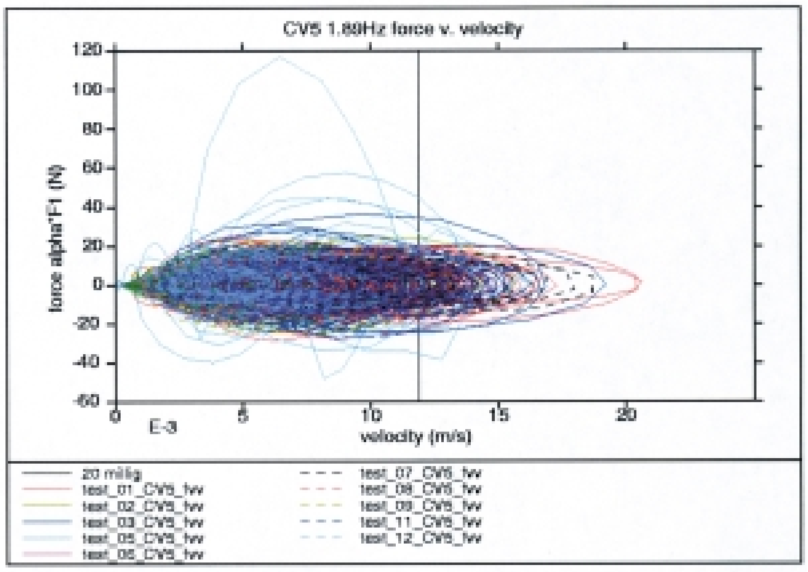
Equations 3 and 4 can be used to convert equation 2 from a discrete sum to the continuous integral:

$$E(y^2) = \int_{\omega=0}^{\infty} H^2 E(\phi^2) E(f_\omega^2) \frac{N}{\sigma} p\left(\frac{\omega - \mu}{\sigma}\right) d\omega \qquad \text{... equation 5}$$

Assuming the pedestrians are uniformly distributed over the whole span and the modeshape varies sinusoidally spatially and has been normalised to unity:

$$E(\phi^2) = 0.5$$

Fig 18. Vertical force versus velocity - all CV5 data



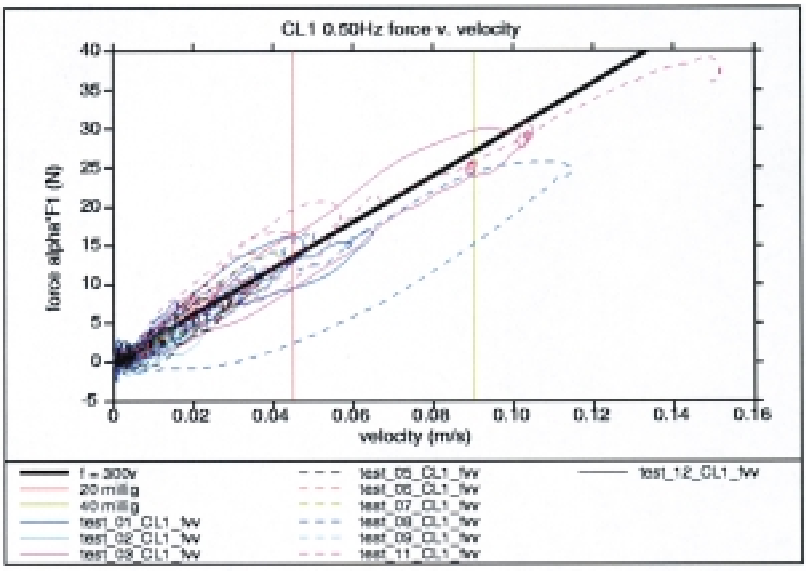


Fig 19. Lateral force versus velocity - all CL1 data

Assuming the force varies sinusoidally temporally:

$$E(f_\omega^2) = 0.5F_\omega^2$$

Substituting these expressions into equation 5:

$$E(y^2) = 0.25\frac{N}{\sigma}\int_{\omega=0}^{\infty} H^2 F_\omega^2 \, p\left(\frac{\omega-\mu}{\sigma}\right) d\omega$$

This is difficult to integrate other than numerically. An approximate solution can be obtained by assuming the response around the natural frequency is dominant, and taking F and p as constant with their values at the natural frequency:

$$E(y^2) \approx 0.25\frac{N}{\sigma} p\left(\frac{\omega_n-\mu}{\sigma}\right) F_{\omega_n}^2 \int_{\omega=0}^{\infty} H^2 \, d\omega$$

This expression includes the standard integral:

$$\int_{\omega=0}^{\infty} H^2 \, d\omega = \frac{\pi\omega_n}{4cK^2}$$

Giving:

$$E(y^2) \approx \frac{\pi N}{16c}\frac{\omega_n}{\sigma} p\left(\frac{\omega_n-\mu}{\sigma}\right)\left(\frac{F\omega_n}{K}\right)^2$$

The corresponding acceleration is approximately:

$$E(a^2) \approx \omega_n^4 E(y^2) \approx \frac{\pi N}{16c}\frac{\omega_n}{\sigma} p\left(\frac{\omega_n-\mu}{\sigma}\right)\left(\frac{F\omega_n}{M}\right)^2$$

Using $F\omega_n = 280N$, $\mu = \omega_n$ and $\sigma = \omega_n/10$ this formula conservatively predicted the vertical responses seen in the Millennium Bridge tests. Closer agreement could be obtained by allowing for the reduction in walking force that occurs with the reduction in walking speed due to increased crowd density.

Pedestrian lateral loading

The objective of the studies reported here was to provide the data needed to solve the vibration problem on the Millennium Bridge. The following sections describe the pedestrian lateral loading theory underlying the solution. While much of what was learnt is applicable to other bridges, aspects of the solution are particular to the Millennium Bridge. The section discussing the general approach (see later) explains why the solution is not necessarily appropriate for other bridges and what else would need to be considered in a generalised design approach.

Lateral walking force coefficient

The tests on the Millennium Bridge showed that the lateral force generated by pedestrians is approximately proportional to the response of the bridge. The correlated force per person can be related to the local velocity V_{local} using a lateral walking force coefficient k:

$$\alpha F_1 = kV_{local}$$

The tests indicated the value of k to be approximately 300Ns/m over the frequency range 0.5 to 1.0 Hz. The i-th person's contribution to the modal force is $\phi_i \alpha F_1$ and the local velocity is related to the modal velocity by $V_{local} = \phi_i V$. The i-th person's contribution to the modal force is therefore:

$$\phi_i \alpha F_1 = \phi_i kV_{local} = \phi_i^2 kV$$

Hence the modal excitation force generated by N people on the span is:

$$Fe = \sum_{i=1}^{i=N}\left(\phi_i^2 kV\right) = kV\sum_{i=1}^{i=N}\phi_i^2 \qquad \text{... equation 6}$$

This loading model has important implications for bridge design, in terms of the damping requirements, the number of users and the effective damping that will be achieved.

Damping requirement for stability

Because the lateral force is proportional to the bridge response, a certain level of damping is needed if the damping force is to exceed the excitation force. Assuming the damping is linear viscous, the modal damping force is:

$$D = CV$$
$$D = cC_{crit}V$$

$$C_{crit} = 2\sqrt{MK} = 2\omega M$$
$$D = 2\omega cM \times V \qquad \text{... equation 7}$$

If the damping force is to exceed the excitation force:

$$D > Fe$$

$$2\omega cM > k\sum_{i=1}^{i=N}\phi_i^2 \qquad \text{... from 6 and 7}$$

$$c > \frac{k\sum_{i=1}^{i=N}\phi_i^2}{2\omega M}$$

Converting to frequency in Hz, the required damping is:

$$c > \frac{k\sum_{i=1}^{i=N}\phi_i^2}{4\pi fM} \qquad \text{... equation 8}$$

This can be evaluated if the modeshape and the distribution of pedestrians is known. Assuming the people are uniformly distributed over the whole span, the number of people in the length dL is:

$$dN = \frac{N}{L}dL$$

The summation can be approximated by the continuous integral:

$$\sum_{i=1}^{i=N}\phi_i^2 = \int_0^N \phi^2 \, dN = \frac{N}{L}\int_0^L \phi^2 \, dL$$

Assuming the modeshape is sinusoidal and normalised to unity:

$$\frac{N}{L}\int_0^L \phi^2 \, dL = \frac{N}{2}$$

Substituting this expression for the summation in equation 8:

$$c > \frac{Nk}{8\pi fM} \qquad \text{… equation 9}$$

Limiting number of people
For a given level of damping, the limiting number of people to avoid instability is:

$$N_L = \frac{8\pi cfM}{k} \qquad \text{… from equation 9}$$

This formula has been derived assuming the pedestrians are uniformly distributed over the whole span and the modeshape is sinusoidal and has been normalised to unity. The numerical coefficient alters in other circumstances, such as in the Millennium Bridge tests where the walkers were confined to parts of the span. Allowing for the distribution of walkers, this approach conservatively predicted the numbers of pedestrians needed to cause the lateral response to increase rapidly in the Millennium Bridge tests.

Effective damping
The amplitude related correlated excitation is equivalent to negative damping, having a critical damping ratio given by:

$$c_e = -\frac{k\sum_{i=1}^{i=N}\phi_i^2}{4\pi fM} \qquad \text{… from equation 8}$$

Assuming that the distribution of pedestrians remains the same, the negative damping varies linearly with N. At N_L the negative damping equals the actual damping, making the effective damping zero. The effective damping is:

$$c_{eff} = c + c_e$$

$$c_{eff} = c\left(1 - \frac{N}{N_L}\right)$$

If the amplitude related correlated force is the only external force, the response of the system will vary exponentially over time such that the response ratio after n cycles is:

$$\frac{A_n}{A_0} = \exp(-2\pi nc_{eff})$$

If the effective damping is positive the response will decay, if negative the response will grow until it is limited by some other effect, such as people deciding to stop walking.

Generalised approach
The Millennium Bridge is expected to get crowded reasonably frequently. The modifications are intended to provide damping sufficient to ensure stability for a crowd density of 2 people/m^2, even though the tests indicated that normal walking becomes difficult at densities above about 1.7 people/m^2. Designing for this extreme density means the bridge should always have significant positive damping, such that the response will remain comfortable. The pedestrian test results support this approach. Although lateral vibrations occurred from the onset, increasing with pedestrian numbers, the acceleration amplitudes remained acceptably small provided the number of pedestrians was less than N_L by a suitable margin.

Designing for an extreme density of pedestrians avoids some of the complications that would need to be researched further before a generalised design approach could be developed.

The theoretical limiting numbers are for people who are walking. In normal use some people would be stood still on the bridge, possibly contributing extra damping. This would have the effect of increasing the limiting numbers that would apply in practice. No advantage has been taken of this effect in designing the modifications to the Millennium Bridge, but it might be appropriate to consider it in the design of other bridges.

It would be reasonable to use lower densities to design bridges which were not expected to get crowded. However the possibility that an increase in the crowd density (either overall or local to where the lateral modeshape was large) would lead to excessive response or instability would need to be considered.

In addition to amplitude related correlation, allowance should be made for correlation due to people falling into step with each other, or responding to visual clues such as the movement of people in front of them. Such correlation will be confined to groups of people and will not extend over much distance. The effect will not be significant on structures designed for use by large numbers of people, such as the Millennium Bridge, but could be on a bridge designed for fewer people.

In addition to the amplitude related correlated force, allowance should be made for an additional random component. The Millennium Bridge is designed for use by large numbers of people, making the random component negligible. Again this would not be the case for a bridge designed for fewer people. One possible way of allowing for the random component of excitation would be to use a modified version of the approach described in the pedestrian vertical loading model for vertical excitation, with the effective damping modified as described in the section on effective damping.

Case studies of other bridges

The experimental results show that the phenomenon of synchronous lateral excitation is not related to the technical innovations of the Millennium Bridge. The same effect could occur on other bridges which have a frequency of less than approximately 1.3 Hz and are loaded by a sufficient number of pedestrians.

The implications of this conclusion are significant for the engineering profession and are particularly difficult to assimilate in the light of the fact that the structure of the Millennium Bridge is so unusual, and that the phenomenon has not been recorded on many other occasions. Other cases have, however, come to light during the research on the bridge. The following examples have been chosen to illustrate the range of structural types on which synchronous lateral excitation has been observed.

Fig 20. Case studies - Groves Bridge, Chester (left) and Birmingham NEC link (right)

Link bridge from National Exhibition Centre to Railway Station, Birmingham

- Built 1978
- Total span 45m, made up of three spans
- Steel truss on unbraced vertical columns
- Sideways movement experienced when large crowds crossed the bridge after events in the Exhibition Centre, eg pop concerts.
- Lateral frequency under full live load 0.7Hz

Fig 21. Auckland Harbour Road Bridge, New Zealand

Groves Suspension Bridge, Chester

- Rebuilt in form of original 1923 structure
- Total span approximately 100m,
- Steel suspension structure, timber deck
- Sideways movement experienced in 1977 during Jubilee river regatta when exceptionally large crowds occupied the bridge to view the river

North section of Auckland Harbour Road Bridge, New Zealand

- Two No. two lane extensions built beside existing road bridge, completed 1965
- Span of north section 190m
- Steel box girder
- Sideways movement experienced in 1975 during a public demonstration when one two lane section was closed to vehicles and crossed by between 2000 and 4000 demonstrators
- Lateral frequency 0.67Hz

The final example is particularly significant because it is a large roadbridge with a conventional structure. The received report of this movement could also be corroborated by a film of the demonstration obtained by Arup which was taken by the New Zealand Television Channel, TVNZ.

In all of the above cases, the phenomenon was not fully researched or analysed, and its occurrence was not widely disseminated within the engineering profession.

Development of retrofit design

General considerations

Unless the usage of the bridge was to be greatly restricted, only two generic options to improve its performance were considered feasible. The first was to increase the stiffness of the bridge to move all its lateral natural frequencies out of the range that could be excited by the lateral footfall forces, and the second was to increase the damping of the bridge to reduce the resonant response.

Lateral stiffening options

Synchronous lateral pedestrian loading is now believed to be possible at any frequency below about 1.3Hz. Therefore, to avoid the possibility of resonant structural response, a minimum target natural frequency of 1.5Hz for any mode with a significant lateral component was set for our studies into stiffening. The most difficult mode to stiffen is the first lateral mode of the centre span, which has a natural frequency of 0.49Hz. To achieve a threefold increase in natural frequency in theory the lateral stiffness must be increased by a factor of nine, if this can be done without increasing the mass at the same time. In practice, any stiffening scheme will also involve increasing the mass so the stiffening factor required is well in excess of tenfold.

Almost all of the stiffness of the main span is derived from the (geometric) tension stiffness of the main cables. Even if the deck were to be fully braced between the edge tubes and the edge tubes made continuous, the natural frequency of the first lateral mode of the centre span would only increase by a few percent. Making the structure a factor of 10 times stiffer cannot be achieved without considerable additional structures, involving very costly works that would also significantly affect the aesthetics of the bridge.

Further, it was found that even once sufficient structure had been added to move the first central lateral mode to above 1.3Hz, there remained lateral-torsional modes at less than 1Hz which could only be removed by increasing the vertical or torsional stiffness of the structure by the same order as the lateral. This would necessitate a further structure, the addition of which would begin to affect the liveload capacity of the existing bridge.

For these reasons a stiffening option was not favoured when it became apparent that a much less intrusive damping-based solution was feasible.

Damping options

Additional damping has often been used to control the resonant responses of lively structures. A number of types of supplemental damping are available:

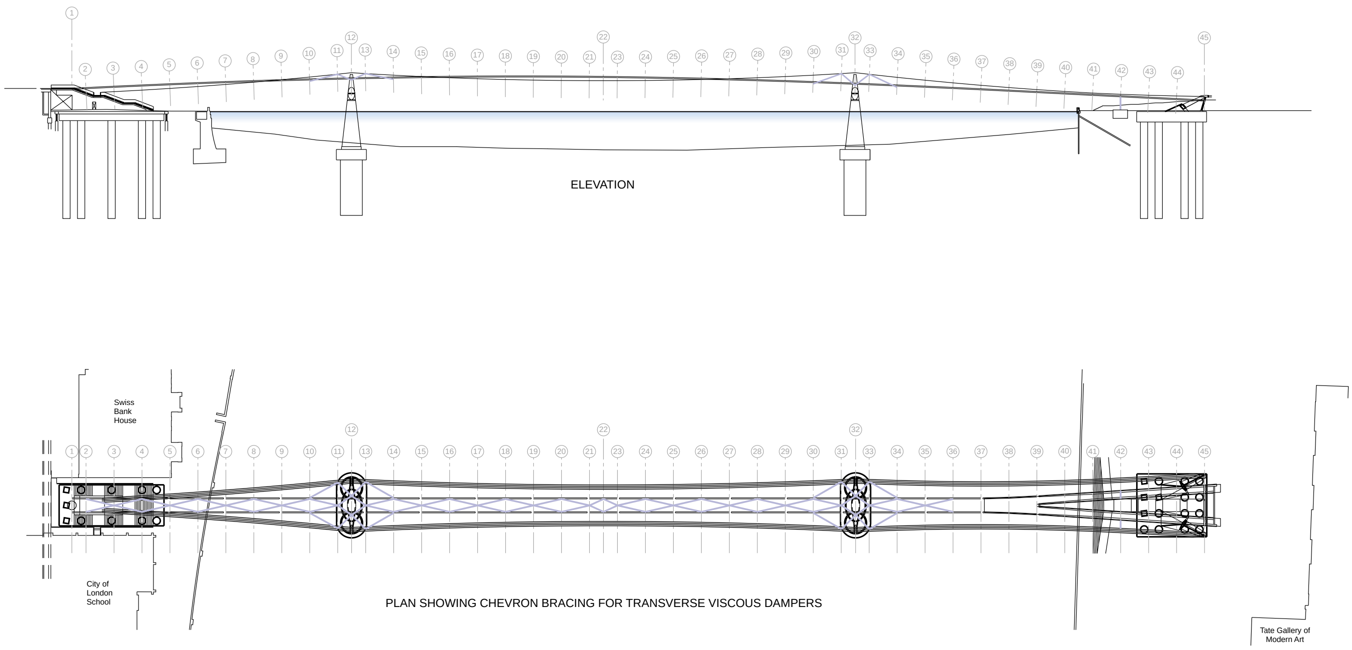
- Tuned mass dampers and tuned slosh dampers
- Visco-elastic dampers, fluid-viscous dampers and friction dampers
- Active control

Dampers in these three listed categories work in distinctly different ways. Tuned mass and tuned slosh dampers are inertial devices that are attached to single points on a structure. Visco-elastic, fluid-viscous and friction dampers are energy-dissipating structural elements that operate on relative movements, and are therefore connected between two points on a structure. Active Control uses computer-controlled powered components of either an inertial or 'stiffness' type to apply forces to the structure to counteract the vibration. All of these options were considered for the bridge.

Description of scheme

The solution that has been developed is based on an assumed load of 2 active walkers/m^2 over the entire bridge. This is considered conservative since our tests showed that at over 1.7 walkers/m^2, the force from pedestrians reduces owing to the dif-

Fig 22. Plan and elevation of bridge showing arrangement of dampers



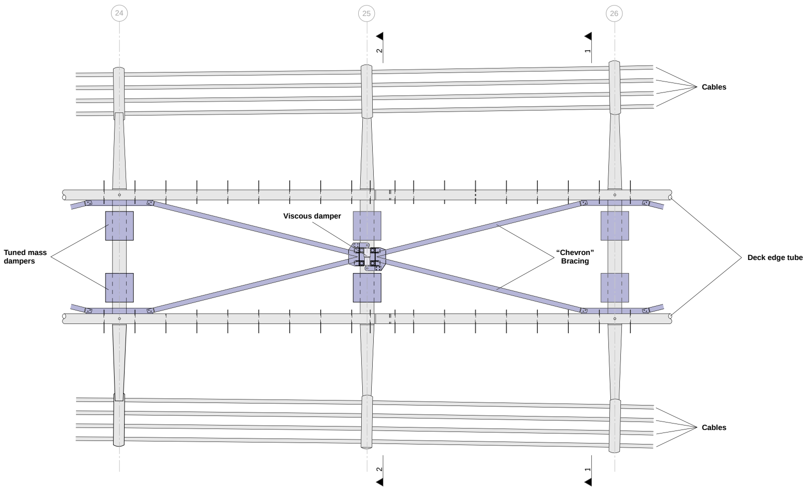


Fig 23. Plan of a typical 16m length of deck showing viscous dampers and tuned mass dampers

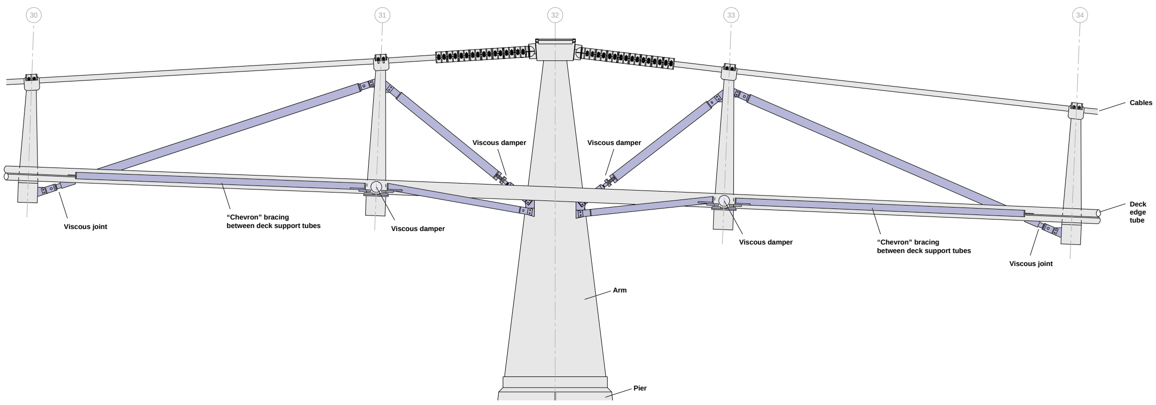


Fig 24. Viscous dampers in plane between cables and deck at piers

ficulty that they have walking in such close proximity.

The priorities in finding a solution were:

- To reduce the movements to an acceptable level
- To minimise the maintenance implications of the modifications
- To maintain the slim appearance of the bridge, in particular to keep the slim elevation.

Design issues

The development of the scheme was driven by a number of specific considerations:

- Achievement of adequate damping levels and small amplitudes of bridge
- Limiting weight added to bridge
- Durability, fatigue and maintenance.

The impact of these issues is addressed briefly in the following sections.

The damping levels required were not known at the early stages of the design, and attention was given to identifying the forms that might lead to the largest achievable damping with minimum impact to the bridge. A broad target was set at 15%–20% of critical damping for the lateral and lateral-torsional modes based on an estimation of the reduction in response that might be required relative to that observed on opening day, and 5-10% for the vertical/torsional modes. The need for vertical and torsional damping is discussed later.

It was found very early in the design process that fluid viscous dampers placed in a 'bracing' pattern under the deck provided an efficient source of damping. The components are light and small, and because they do not need frequency tuning, can provide damping in several modes at once. The main initial concern was whether they would operate in a satisfactory manner at small amplitudes, given the influence of friction in the seals, compressibility of the damping fluid and possible 'play' in the bearings and connecting structure. It was considered essential that the required damping should be achieved when the acceleration response of the bridge was less than 20 milli-g. For modes at 1Hz this corresponds to a peak span deflection of only 10mm, and so the strokes of the individual dampers would be of the order of a millimetre during normal operation.

A number of manufacturers were able to demonstrate that their dampers would operate at amplitudes of less that 0.5mm. The damper selected for the final scheme is a low friction fluid-viscous damper manufactured by Taylor Devices Inc. This is a special precision damper, originally developed for space applications where zero friction and zero fluid (and vapour) leakage were demanded. These attributes are achieved by the incorporation of a metal concertina form of seal between the body and the pistonrod/actuating rod. Although low friction is advantageous in this application, the decision to adopt this device type was ultimately made on the grounds of its maintenance-free operation.

Tuned mass dampers were also examined initially, as these are a 'conventional' fix for bridges with excessive vibration. A number of limitations became apparent very quickly. There are in all eight modes (two lateral, four vertical, two torsional) that would require damping in the centre span alone. Every mode needs its own mass tuned to the modal frequency. Large masses would be needed and therefore the total weight added to the bridge would be high. Although the mass required might not be a concern for strength, the deflection of the main span would be considerable, and navigation clearances would be compromised.

In the final design some tuned mass dampers are used since there is no suitable load-path in which to introduce viscous dampers for the vertical modes of the main span.

Horizontal damping

Horizontal damping is provided primarily by viscous dampers. A total of 37 viscous dampers are to be installed. The majority of these dampers are to be situated beneath the bridge deck, on top of alternate transverse arms (every 16m). Each end of the viscous damper is connected to the apex of a steel V brace, known as a chevron. The apex of the chevron is supported on roller bearings that provide vertical support but allow sliding in all directions. The other ends of the chevron are fixed to the neighbouring transverse arms. In this way the lateral modal movement over 16m is mobilised at each damper. This means that the strokes of the dampers are greater than if for example they were connected directly as diagonals over an 8m bay.

Where possible the viscous dampers are connected to fixed points such as the piers and the ground. This makes the dampers more efficient since the movement is transferred

Fig 25. Viscous dampers connected to ground at the south abutments

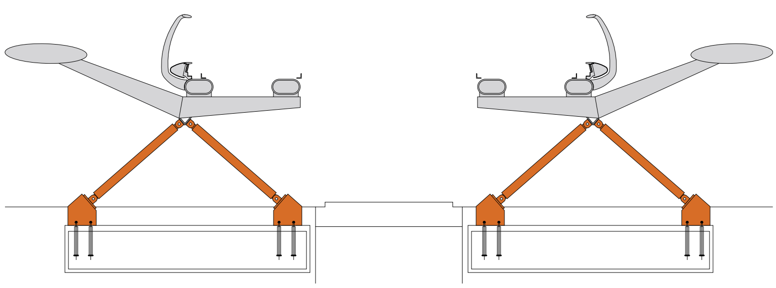


Fig 26. Installation of prototype viscous dampers

directly into the damper stroke rather than via a structure which is itself subject to deformation. There are viscous dampers in the plane between the cables and the deck at the piers. These provide vertical and horizontal damping. In addition, a pair of dampers are located to each side of the approach ramp on the south abutment. These provide damping primarily for the lateral and lateral-torsional modes on the south span.

On the assumption that the effective pedestrian input force is linearly related to the velocity of the bridge, the greatest damping requirement is for the lowest frequency mode. The first central lateral mode has a frequency of 0.49Hz and this results in a damping requirement of over 20% of critical damping for the loading assumed. In order to achieve this value of damping it has been necessary to add 4 No. pairs of lateral tuned mass dampers to the central span. Each of these dampers comprises a mass of 2.5t hung by a double pendulum at each corner. A steel paddle is attached to the mass and this paddle sits in a pot of viscous fluid which itself is attached to the transverse arm of the bridge. Damping is therefore generated when the mass moves relative to the transverse arms.

Vertical damping

The Millennium Bridge did not respond excessively in the vertical direction on the opening day, and, as shown above, our measurements during crowd walking tests indicate that vertical force input from pedestrians is random rather than correlated. Some researchers have nevertheless suggested that synchronous pedestrian vertical loading is also possible and has been observed elsewhere. The possibility was therefore considered that, once excessive lateral motions had been suppressed by provision of lateral damping, pedestrians might find the vertical motions of the bridge more noticeable, and synchronise with them.

Stiffening the bridge vertically to raise the first mode frequency by a factor of five from 0.5Hz to over 2.5Hz to avoid resonance with vertical walking forces is even more difficult than achieving the threefold increase required in the lateral frequency. A damping approach was therefore also adopted as a precaution against possible excessive vertical motions.

Vertical damping is provided primarily by vertical tuned mass dampers. A total of 26 pairs of vertical tuned mass dampers will be installed over the three spans of the bridge. These comprise masses of between 1 and 3t supported on compression springs. As with the lateral tuned mass dampers, viscous damping is provided by the presence of a paddle connected to the mass and suspended in a pot of viscous fluid connected to the transverse arm.

The tuned mass dampers are situated on top of the transverse arms beneath the deck. They are arranged along the length so that they are at or close to the antinodes of the modes that they are damping.

Maintenance

The frequency of inspections for the modifications is the same as that for the original bridge, and follows the recommendations set out by the Highways Agency.

The viscous dampers and the tuned mass dampers are located below sections of the deck that can be removed from above. This enables inspection to take place largely from the deck. The viscous dampers are covered by a 35-year warranty for their internal parts. They require no maintenance during this period, other than repainting which is at the same frequency as the existing structure.

The tuned mass dampers are warranted for a first maintenance after 10 years provided that the 2-yearly spot inspections recommended by the Highways Agency are carried out. The viscous damper unit on the tuned mass dampers can be accessed from above and lifted out to be maintained or replaced.

Quantification and optimisation of the scheme

The benefit of introducing a scheme of dampers cannot be evaluated using conventional structural engineering software. The main viscous dampers and elements of the tuned mass dampers are energy-dissipating components which respond to loading with a force proportional to relative velocity between their ends rather than relative displacement. There is an optimum value for the damping rate of each component. If the element is too 'soft' it will clearly not dissipate much energy, but also if it is too 'stiff', then the deflection across it will be small and it will not dissipate much energy either. The scheme was optimised by examination of the best locations at which to place dampers, their optimum damping rates, the appropriate stiffnesses for the additional chevron elements and the best masses, damping rates and tuning frequencies for the tuned mass dampers. The design development and optimisation was undertaken using the MSC-NASTRAN finite element program. The compressibility of the damper fluid was modelled as a series spring in NASTRAN. However, the non-linear effects of friction and play in the bearings could not be modelled. These effects were analysed separately using an approximate iterative method and were shown to only be significant at very small bridge amplitudes.

The optimisation study examined the complex modes (the modeshapes are altered by the viscous components) of the damped system and the frequency response functions of the bridge under distributed lateral and vertical dynamic pedestrian loading patterns.

Validation by testing

Prototype installation: In order to demonstrate conclusively that the proposed damping scheme would work as predicted, a prototype installation was made on the bridge and tested in December 2000. Two viscous dampers, each connected to the structure by two pairs of chevrons, were installed on the central span (at gridlines 15 and 29) for the prototype test, together with

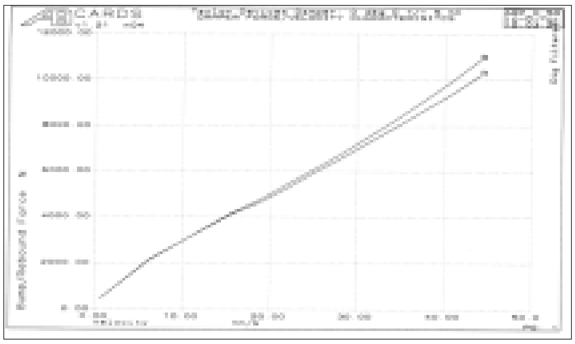


Fig 27. Force-Velocity test results at 0°C for the viscous damper used in the December 2000 tests

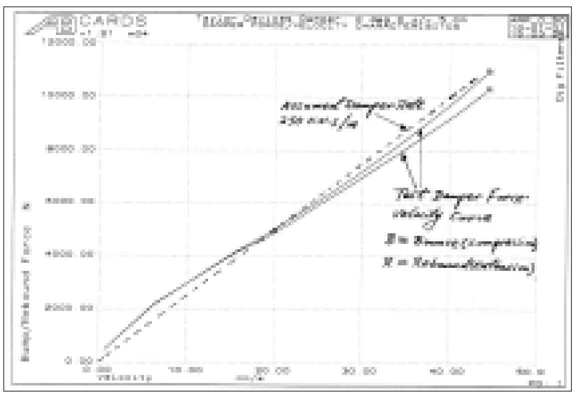


Fig 28. Force-Velocity test results with assured damper rate of 250kNs/m shown

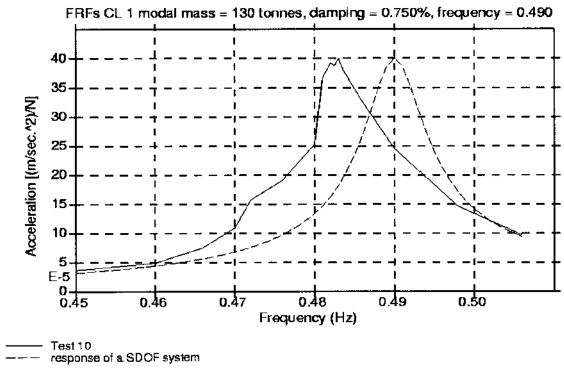


Fig 29. Curve fitting of frequency response function for the 1st lateral mode on the centre span

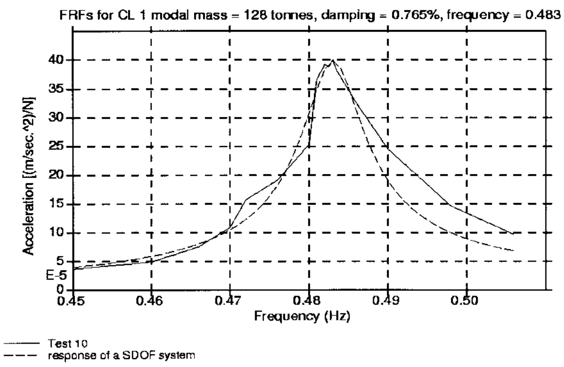


Fig 30. Curve fitting for 1st lateral mode on the centre span - natural frequency of SDOF system reduced

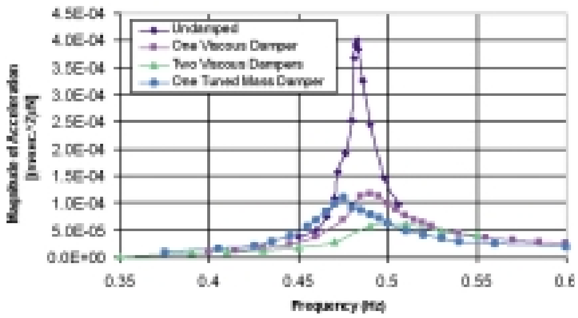


Fig 31. Comparison of measured FRFs of the undamped and damped bridge

a lateral double pendulum tuned mass damper at midspan.

The viscous dampers, together with their corresponding clevises, spherical bearings and pins, were supplied by Taylor Devices Inc. In view of the timescale for manufacture, these units were of a conventional design, and did not have the frictionless metal bellows seals

Independent tests were performed on the viscous damper units at temperatures of 0° C and at ambient (15° C). The force-velocity test results are given in Fig 27. The measured friction is about 0.6kN and the viscous rate 215kNs/m. For the purposes of predictive analysis for the bridge with the prototype dampers in place an equivalent linear damper rate of 250kNs/m has been taken. This is compared with the test results in Fig 28. It is seen that this results in a reasonable representation of force over the range of principal concern.

The configurations of viscous dampers tested were:

(i) both viscous dampers fitted
(ii) only the GL15 damper (northern end of the centre span) fitted
(iii) no viscous dampers fitted.

In the cases where the viscous damper(s) were removed, the whole damper assembly, including the clevis mounting plates, was removed from the chevrons.

The TMD frequencies and damping were determined by analysing the recorded acceleration time histories of the TMD mass when excited to resonance by human action and then allowed to decay freely. In these tests, the TMD was fixed to the ground.

Test procedure: The bridge was excited in the lateral direction using a 1000kg steel mass placed on rails on the deck, and driven from side to side at the required frequency by a hydraulic ram. The force input to the structure could be accurately calculated, as could the resultant steady state acceleration. In this way the damping for each mode could be calculated.

Assessment of modal properties: The modal properties were estimated by examination of the measured frequency response functions (FRF) of the bridge with the various configurations of dampers in place. A formal multi-degree of freedom curve fitting procedure was used initially, but during this process some difficulties in curve-fitting techniques became apparent, principally on account of the amplitude–dependent non-linear behaviour of the bridge. This may be illustrated by the assessment of mode CL1, the first lateral mode of the centre span, from its midspan FRF. These measurements should be well represented by a single degree of freedom system, since the next lateral mode in the main span having significant modal mobility at midspan has about three times the frequency. Single degree of freedom curve fitting concentrated initially on matching the responses away from resonance; since the response is not sensitive to damping in this region, the modal mass and natural frequency can be adjusted in order to match the FRF shape to both sides of the resonance. The damping is then varied in order to match also the general height and bandwidth of the frequency response curve.

Typical results are shown in Fig 29. It can be seen that in order to match the FRF away from resonance a natural frequency higher than that indicated at the measured resonance peak must be selected. This is consistent with the behaviour of a non-linear system that becomes more flexible at higher amplitudes. In this case this may be due to slippage at frictional joints etc. only occurring at higher amplitudes. The softening of the bridge at higher vibration amplitudes is manifested by the 'bend' of the measured FRF curve towards the lower frequencies, as shown in Fig 29. The satisfactory fit to the shape and the peak response of the FRF curve at resonance is illustrated in Fig 30 in which the natural frequency of the SDOF system has been reduced.

Results: The results of the investigation are summarised below. For each modal property, a range has been given to reflect the amplitude dependent non-linear behaviour of the bridge due to softening at larger vibration amplitudes.

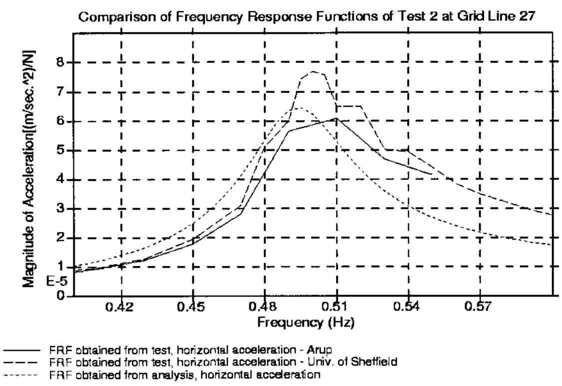


Fig 32. Predicted and measured FRFs for mode CL1 with two viscous dampers in place

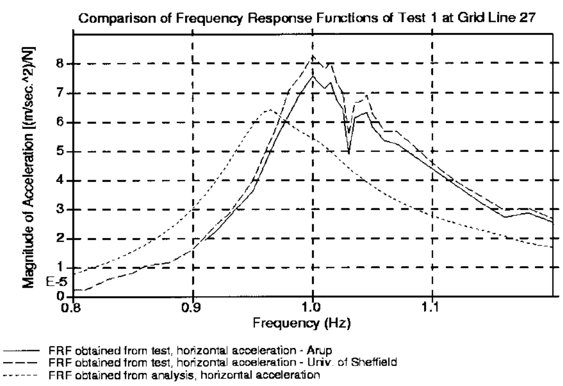


Fig 33. Predicted and measured FRFs for mode CL2 with two viscous dampers in place

Table 1 shows the results of the modal survey for the bridge with two viscous dampers fitted and with the tuned mass damper inactive.

Table 2 shows the results of the modal survey for the bridge with no viscous dampers fitted and with the bi-directional tuned mass damper active in both directions.

Figure 31 shows the effect on the measured FRF of CL1 of adding one, and then two viscous dampers and the lateral TMD. It is seen that just two viscous dampers make a substantial reduction to the response of the bridge.

Back-analysis of structure

The measurements on the undamped structure confirmed that the natural frequencies of the key modes were close to those assumed in the original analysis. The difference may be due to differences in the cable tensions, the masses and secondary stiffnesses between the actual structure and the models. For the purpose of this analysis, the cable tensions in the model were adjusted to ensure that the natural frequencies matched as closely as possible.

Table 3 compares the natural frequencies obtained with the measured values.

The effect of adding the various prototype dampers was assessed using NASTRAN complex mode and frequency response analyses, as had been done for prediction of the performance of the final complete scheme. Figs 31 and 32 show the predicted and measured FRFs for modes CL1 and CL2 with two viscous dampers in place. The agreement for CL2 is excellent and for CL1 is within 10%.

Validation of final scheme

The tests demonstrated that the retrofit scheme will provide the predicted and required levels of damping. In particular:

(1) The prototype arrangement of viscous dampers connected to the underside of the deck with steel chevrons provided levels of damping in accordance with analytical predictions. The tests also showed that the prototype dampers used (with standard seals and friction) performed satisfactorily down to the small amplitudes required to control responses to acceptable levels. The metal bellows dampers to be provided in the final scheme should perform better than the standard seal units used in these tests.

(2) The lateral and vertical tuned mass dampers performed in accordance with analytical predictions. The single axis double pendulum lateral damper performed in a satisfactory manner. The importance of achieving a sufficiently accurate TMD frequency is being addressed in the final scheme, which allows for a suitable level of mis-tuning. It should also be noted that the final scheme TMD performance is significantly less sensitive to TMD tuning than in the prototype configuration, due to the greater TMD mass and damping levels.

Table 1: Comparison of modal properties of the bridge with two viscous dampers

Mode description	Frequency (Hz)	Damping (%)	Modal mass (t)
CL1	0.50 - 0.52	4.1	130 to 133
CL2	1.0 - .03	3.8 to 3.9	155

Table 2: Comparison of modal properties of the bridge with two viscous dampers

Mode description	Frequency (Hz)	Damping (%)	Modal mass (t)
CL1	0.478-0.484	2.0	128

Table 3: Comparison of modal properties obtained from tests and analysis

MODE	Measured frequency (Hz)	Analysis frequency (Hz)	Measured modal mass (t)	Analysis modal mass (t)
CL1	0.48	0.49	128 - 130	144
CL2	0.95	0.94	145 - 148	150
CV3	1.15	1.18	155	150
CV4	1.54	1.56	140	138
CV5	1.89	1.93	135	146
CV6	2.32	2.30	135	132
SL1	0.80	0.80	160	181
NL1	1.03	1.00	Not available	113

Fig 34. Photomontage of bridge with dampers installed

Modification works installation

Cleveland Bridge UK has won the tender to be the contractor for the modification works project. The viscous dampers are being fabricated by Taylor Devices in America, and the Tuned Mass Dampers are being fabricated by Gerb Schwingungsisolierungen in Germany.

Work began on site at the beginning of May 2001 and will be complete by the end of 2001.

Discussion of results and implications

Implications for the engineering profession

The British Standard on bridge live loading, BD 37, has incorporated a clause with regard to the phenomenon of synchronous lateral excitation in its latest draft, BD 37/01. The Institution of Structural Engineers' Standing Committee on Structural Safety (SCOSS) has included a section warning of this phenomenon in its recently issued 13th report[2].

Arup is providing consultancy on a number of bridges around the world which might be susceptible to this phenomenon. Arup has also written to the following organisations informing them of their findings:

- DETR
- Highways Agency
- IABSE
- Institution of Civil Engineers
- Institution of Structural Engineers

Conclusions

The following conclusions can be drawn as a result of the research and analysis that has been carried out with regard to the phenomenon of synchronous lateral excitation:

(1) The phenomenon of synchronous lateral excitation is not linked to the technical innovations of the Millennium Footbridge.

(2) The same phenomenon could occur on any future or existing bridge with a lateral frequency below approximately 1.3Hz, loaded with a sufficient number of pedestrians.

References

1. AASHTO 'Guide Specification and Commentary for Vessel Collison Design of Highway Bridges': Vol. 1 Final Report, February 1991
2. Pavic, A., Reynolds, P., Wright, J. *Analysis of frequency response functions measured on the Millennium Bridge, London*, 1st Draft, ref CCC/01/0082A, University of Sheffield, January 2001
3. *Structural Safety 2000-01.* Thirteenth Report of SCOSS, Institution of Structural Engineers, May 2001